

Assessment of the South African sardine resource using data from 1984-2010: initial results for a two stock hypothesis

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Summary

The sardine assessment is extended to consider three hypotheses for the presence of two (west and east) stocks separated at Cape Agulhas. The first hypothesis of two completely separated stocks is dismissed, as it implies that an unrealistically low proportion of the recruits for the eastern stock are detected in the recruitment survey. The second hypothesis of movement of recruits from west to east stocks, the extent of which varies from year to year, is more successful in terms of matching the data for realistic values of detectability of eastern recruits in the recruitment survey. However the proportion moving from year to year is only weakly correlated with high biomass or recruitment in the western stock that year. The third hypothesis is a variant of the second for which the data are less informative because the “dividing line” between west and east recruits is taken to be unknown. This (in the one of several possible forms implemented thus far) produces similar results to the second hypothesis. It is proposed that because of the limited time available now to complete the OMP revision, and further issues still to be resolved relating to future projections under two stock hypotheses in relation to the distribution of catching, that future work in finalising this OMP process be restricted to the second hypothesis (specifically the variant which assumes movement from 1994 onwards onwards when the recruit survey was extended further eastwards).

Introduction

As part of the process of updating the assessment of the South African sardine resource, a two sardine stock hypothesis is to be tested. This hypothesis consists of a “west” stock distributed west of Cape Agulhas and an “east” stock distributed east of Cape Agulhas. Initial results showed that a hypothesis of two discrete stocks was not incompatible with the data (de Moor and Butterworth 2009), although the multiplicative biases estimated to be associated with the May hydroacoustic survey in the east was questionable. A hypothesis of two mixing stocks could not be adequately explored with the model at that time due to the lack of ageing data.

The updated operating model for the South African sardine resource is detailed in Appendix A of de Moor and Butterworth (2011) and the data used in this assessment are listed in de Moor *et al.* (2011). All key changes to the model described in de Moor and Butterworth (2011) also apply to the model used for the two stock hypothesis. A glossary of terms used in this model is repeated from de Moor and Butterworth (2011) in the Appendix of this document for ease of reference.

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This document details some initial results relating to a two stock hypothesis. The document is arranged such that a hypothesis is described, and then results pertaining to that hypothesis are shown and discussed. In all results given in this document, adult natural mortality is fixed at $\overline{M}_{ad}^S = 0.6$, with no random effects, i.e. $\eta_y^{ad} = 0$ for all years. This is to force initial results from the two stock hypothesis to be broadly comparable to the single stock hypothesis, rather than to allow extreme deviations from 0.6 yr⁻¹.

Hypothesis 1: Two Discrete Stocks

The first hypothesis tested is a two discrete stock hypothesis. In this case the proportion of 1 year olds moving from the west stock to the east stock at the beginning of November is set to zero, i.e. $move_y = 0$ for all $y = 1984, \dots, 2010$. Three alternatives were tested in this case:

- i) Prior distributions for the bias parameters as described in Appendix A of de Moor and Butterworth (2011), excluding the prior on the recruitment residuals
- ii) As for i), except with $k_{cov}^S \sim U(0,3)$
- iii) As for i), except including the prior on the recruitment residuals.

Results

The population model fits to the time series of abundance of November 1+ biomass and May recruitment are shown in Figures 1 and 2, respectively. Although the fit to the November 1+ biomass is good, the fit to the west stock May recruitment is very poor for alternatives i) and iii), particularly during the late 1990s and early 2000s.

The estimated parameter values and other key outputs are listed in Table 1 together with the individual contributions to the objective function at the posterior mode. The assumption is made that the survey covers the full distribution of the 1+ sardine (west and east) in November. The November hydroacoustic survey estimate is, however, still estimated to be a relative (under-) estimate due to biases associated with the hydroacoustic survey (see Appendix B of de Moor and Butterworth (2011)). For alternative i) $k_{cov}^S = 1$ implies that the May hydroacoustic survey is able to cover the full distribution of recruits from the west stock. This does not seem unreasonable. However, $k_{covE}^S = 0.18$ implies that the May hydroacoustic survey is only able to survey 18% of the recruits from the east stock. The fit to the May recruitment time series is substantially improved when the upper bound on the prior on k_{cov}^S is increased, removing the limitation imposed on alternative i) that the May survey does not detect a greater proportion of recruits than the November survey does of 1+ sardine. This restriction is reasonable given

the near-shore distribution of recruits, outside of the range covered by the survey (though an estimate is still made for this area), and furthermore the fact that not all the recruitment for the year has taken place by the time of the survey. The recruit survey estimates can be better mimicked by the model when $k_{cov}^S = 2.3$ (alternative ii), Table 1) implying that the May hydroacoustic survey estimate of recruits overestimates the number of west stock recruits by more than double!

There is little difference in the results from i) and iii) except that in iii) the variance about the stock recruitment relationship, $(\sigma_{j,r}^S)^2$, is more accurately estimated (Table 1).

In all cases the model substantially overpredicts the number of recruits estimated from the surveys in the east stock in May 2001 and 2003.

Figures 3 and 5 show the fits to the time series of survey and commercial proportion-at-age data, respectively for alternative i). The model estimated survey and commercial selectivities at age are plotted in Figures 4 and 6, respectively. As for the single stock hypothesis, it is clear that the upper and lower bounds of the uniform prior distribution are constraining the survey selectivities at ages 1 and 2. The model generally over-predicts the quarterly commercial proportions-at-age 2 and under-predict the proportions-at-age 3. This may be related to the relatively high commercial selectivity estimated for age 2 and relatively low commercial selectivity estimated for age 3. This will be further investigated with a model that is able to fit the hydroacoustic survey data satisfactorily.

The model predicted November spawner biomass and recruitment at the posterior mode are plotted in Figure 7, together with the model estimated hockey-stick stock-recruitment curve for alternative iii).

Note that for all these three scenarios, the results do not have a positive definite Hessian, implying the models may not have adequately converged to the posterior mode.

Implications

If the assumption that the November survey covers the full distribution of the west and east stocks is made, while the May survey covers at most the full distribution of the west and east stocks, i.e. $k_{cov}^S \sim U(0,1)$, the proposed hypothesis of two discrete sardine stocks is not able to adequately fit the time series of May recruitment from the west stock. In order to obtain an improved fit to the time series of recruitment from the west stock, the model estimates that the May survey overestimates the number of recruits in the west stock by more than double. In both cases the model estimates the ratio of $k_{j,N}^S : k_{j,r}^S$

for the east stock to be 18 – 21%, with a ‘coverage’ of the east stock recruits being only 9-18% of that of the west stock recruits. The acceptance of such a hypothesis would imply that only about one fifth of the recruits of the east stock is surveyed during the annual May hydroacoustic survey. On the basis of these estimates of the coverage bias in the hydroacoustic surveys, this hypothesis is considered implausible.

Hypothesis 2: Two Stocks With Eastward Migration of Recruits

The second hypothesis tested is an extension of the first to allow for one-way migration of recruits from the west stock to the east stock in November, i.e. as they are modelled to turn 1 years old. As the time series of hydroacoustic survey estimates May recruitment for the east stock begins in 1993, the proportion of west stock recruits that annually migrate to the east stock, $move_y$, can be estimated directly only over 1994-2010. Two alternatives were tested in this case:

- i) $move_y$ is estimated for 1994-2010 and $move_y = 0$ for 1984-1993
- ii) As for i) but $move_y$ for 1984-1993 is set at the average of that estimated between 1994 and 2010.

This alternative is based on the assumption that if migration occurred during the latter part of the time series, there may have also been some migration during the early part of the time series

In all alternatives the prior on the recruitment residuals is excluded from the objective function. There appears to be some confounding of the estimation of the movement parameters and that of the residuals about the stock recruitment curves such that once this prior is included in the objective function, a low variance about the stock recruitment curve is estimated, but a poor fit to the time series of survey estimates results.

Results

Estimating migration of west stock recruits to the east stock between 1994 and 2010 only, with no migration before that, results in a substantial improvement to the fit to the May recruitment data, though with a slightly poorer fit to the November 1+ biomass data, (Table 1, Figures 8 and 9). The coverage of the recruits in the east stock in comparison to those in the west stock, k_{covE}^S , is estimated to be 50% (Table 1), with substantial eastward migration of recruits in 1995 and over 1999-2003 (Figure 10). The residuals from the fit of the model to the May recruitment data in the west stock are still mostly positive, though much better than for Hypothesis 1 of two discrete stocks.

A worse overall fit to the data is obtained under alternative ii) (Table 1, Figures 8 - 10).

Implications

Hypothesis 2 results in a substantially improved reflection of the data compared to Hypothesis 1, with k_{covE}^S estimated to have a more realistic value. Note that k_{cov}^S is still estimated to be at its maximum of 1. Further work would need to be undertaken with this proposed hypothesis if it is to be taken forward to try to correct the almost-all positive residuals in the model fit to west stock recruitment between May 1992 and 2010.

When migration of recruits from the west stock to the east stock was initially proposed, a suggestion was made that such migration may be linked to high biomass or recruitment in the west stock. Figure 11 plots the estimated proportion of west stock recruits migrating to the east stock in November against the model predicted November 1+ biomass of the former year, May recruitment of the current year and November 1+ biomass of the current year. Although some relationship is evident with higher proportions moving eastwards at higher abundances in some years, there are also years with high abundance where almost no migration occurs. The correlation coefficients for the linear relationships shown in Figure 11 are all rather low (0.40, 0.35 and 0.11). This feature is evident for movement expressed in absolute as well as in proportional terms, as can be seen when considering the recruit biomass or numbers which are modelled to move from the west stock to the east stock against the model predicted November 1+ biomass of the former year, May recruitment of the current year and November 1+ biomass of the current year (Figure 12).

If some form of this proposed hypothesis were to be accepted for use in simulation testing OMP-12, future migration from the west to east stocks will need to be modelled. This would be relatively simple if a strong relationship between west stock 1+ biomass or recruitment had been found. In that case the future west stock 1+ biomass or recruitment would closely determine the amount of eastwards migration. In its current form, the model reflects only a weak relationship. Thus future movement modelled would have very high variability about such linear relationships, ranging from zero to near 100%. The implications of that are not immediately clear and will be evident only once projection simulations are conducted, but given this high variability, it is unlikely that any OMP could benefit in terms of performance by assuming some knowledge of the relationship.

Hypothesis 3: Two Stocks With Mixed Recruitment

The third hypothesis tested is based on the assumption that the split between west stock and east stock recruits is not clear and does not necessarily correspond to west and east of Cape Infanta. In this hypothesis, the combined model predicted recruitments of the west and east stocks are fitted to a single time series of recruitment estimates from surveys. Ideally this time series should extend as far east as possible. However, due to the number of years in which the May survey did not extend east of Cape

Infanta, an initial version of this hypothesis uses the time series of recruits estimated by the May survey to Cape Infanta only (i.e. the data formerly considered to be west stock recruits only).

Results

The model results are almost identical to those under Hypothesis 2, alternative i). The contributions of the recruits from the east stock to the total number of recruits is small in absolute terms (Figure 13), although estimated to be larger than those for Hypothesis 2 (Figure 14). The estimated percentage of recruits from the east stock does, however, increase to as much as 50% in 2005 and is above 30% in 9 out of 26 years.

Implications

This implementation of Hypothesis 3 is approximate, as strictly the associated assessment should include making allowance in some way for how past juvenile catches are split between the west and east stocks. Of more concern though is how that split is to be simulated for future projections, depending on different assumed distributions for that catch. Furthermore Hypothesis 3 could be seen as a form of Hypothesis 1 (completely separated stocks) as well as of Hypothesis 2 (some west to east movement), arising in each case from an inability to define a boundary between the recruits from both stocks.

Given the very limited time left available for OMP development, and the time-consuming complexities of taking the various different possible forms of Hypothesis 3 forward at this time, and further the similarities in assessment results under this hypothesis to those under Hypothesis 2, alternative i), it would seem best to focus further two stock work on the latter which is providing a reasonable representation of available data.

References

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- de Moor, C.L., and Butterworth, D.S. 2011. Assessment of the South African sardine resource using data from 1984-2010: further work towards a base case single stock hypothesis. Department of Agriculture, Forestry and Fisheries Document FISHERIES/2011/SWG-PEL/67. 33pp
- de Moor, C.L., Coetzee, J., Durholtz D.,Merkle, D., and van der Westhuizen, J.J., 2011. A final record of the generation of data used in the 2011 sardine and anchovy assessments. Department of Agriculture, Forestry and Fisheries Document FISHEREIS/2011/SWG-PEL/51. 31pp.

Table 1. Key model parameter values and model outputs estimated at the joint posterior mode for the two proposed hypotheses: 1) two discrete stocks and 2) two stocks with movement of recruits from west to east. Values fixed on input are given in **bold**. Numbers are reported in billions and biomass in thousands of tonnes.

Alternative	Hyp I			Hyp II		Alternative	Hyp I			Hyp II	
	i)	ii)	iii)	i)	ii)		i)	ii)	iii)	i)	ii)
Objective function	147.75	139.19	297.53	132.13	170.88	$S_{j,1}^{survey}$	1.10	1.10	1.10	1.10	1.10
$-\ln L^{Nov}$	28.88	33.77	28.55	33.95	46.62	$S_{j,2}^{survey}$	0.90	0.90	0.90	0.90	0.90
$-\ln L^{rec}$	47.43	32.42	48.36	23.48	51.14	$S_{j,3}^{survey}$	0.92	0.91	0.92	0.94	0.94
$-\ln L^{sur propa}$	0.45	0.48	0.45	0.37	0.37	$S_{j,4}^{survey}$	0.90	0.90	0.90	0.90	0.90
$-\ln L^{com propa}$	1.86	2.09	1.87	2.26	1.71	$S_{j,5+}^{survey}$	1.04	1.05	1.04	1.02	1.02
$-\ln L^{coml propl min}$	2.99	2.96	2.99	3.05	3.05	$S_{q,1}$	0.05, 0.99	0.02, 1.88	0.05, 0.99	0.08, 0.52	0.23, 0.49
$-\ln L^{coml propl}$	46.63	46.57	46.61	46.89	47.63	$S_{q,2}$	1.21	1.07	1.21	1.35	1.46
$-\ln L^{sur propl min}$	0.41	0.40	0.41	0.40	0.39	S_3	0.18	0.10	0.18	0.52	0.50
$-\ln L^{sur propl}$	6.76	6.92	6.75	6.92	6.62	S_4	1.00	1.00	1.00	1.00	1.00
$-\ln(priors)$	-6.29	-5.58	142.91	-3.82	-5.30	S_{5+}	1.98, 1.84	1.97, 1.76	1.98, 1.84	1.84, 1.47	1.37, 1.42
\bar{M}_j^S	1.0	1.0	1.0	1.0	1.0	$N_{1,1983}^S$	1.427	2.831	1.428	1.206	0.995
\bar{M}_{ad}^S	0.6	0.6	0.6	0.6	0.6	$N_{2,1983}^S$	0.007	0.010	0.007	0.007	<0.001
k_{ac}^S	0.80	0.63	0.80	0.77	0.81	\bar{B}_{Nov}^S ¹	596	665	595	649	582
k_{cov}^S	1.00	2.35	1.00	1.00	1.00	K_1^S	1190	1414	1161	2159	1816
$k_{cov E}^S$	0.18	0.09	0.18	0.50	0.49	K_2^S	123	95	123	78	63
$k_{j,N}^S$	0.80	0.63	0.80	0.77	0.81	a_1^S	20	20	20	37	31

¹ OMP-04 and OMP-08 were developed using Risk defined as “the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years”. Here the combined west and east stock 1+ biomass is reported.

Table 1 (continued).

Alternative	Hyp I			Hyp II		Alternative	Hyp I			Hyp II	
	i)	ii)	iii)	i)	ii)		i)	ii)	iii)	i)	ii)
$k_{1,r}^S$	0.80	1.47	0.80	0.77	0.81	a_2^S	2.0	1.6	2.0	1.3	1.0
$k_{1,r}^S$	0.14	0.13	0.14	0.38	0.40	b_1^S	221	414	216	303	181
$k_{1,r}^S/k_{1,N}^S$	1.00	2.35	1.00	1.00	1.00	b_2^S	13.5	14.1	11.6	12.0	21.0
$k_{2,r}^S/k_{2,N}^S$	0.18	0.21	0.18	0.50	0.49	$\sigma_{1,r}^S$	0.41	0.41	0.99	0.41	0.41
$(\lambda_{1,N}^S)^2$	0.08	0.12	0.08	0.13	0.14	$\sigma_{2,r}^S$	0.41	0.41	1.17	0.41	0.41
$(\lambda_{2,N}^S)^2$	0.01	0.05	0.01	0.04	0.09	L_∞	19.8	19.6	19.8	19.9	19.8
$(\lambda_{1,r}^S)^2$	0.53	0.17	0.53	0.09	0.09	κ	0.58 (k*L fixed)	0.59	0.58	0.58	0.58
$(\lambda_{2,r}^S)^2$	0.50	0.48	0.50	0.25	0.29	t_0	-1.71	-1.71	-1.71	-1.71	-1.71
						ϑ_1	0.29	0.29	0.29	0.27	0.28
						ϑ_2	0.11	0.12	0.11	0.11	0.10
						ϑ_3	0.38	0.70	0.38	0.35	0.31
						ϑ_4	0.08	0.07	0.08	0.06	0.07
						ϑ_{5+}	0.05	0.06	0.05	0.05	0.06

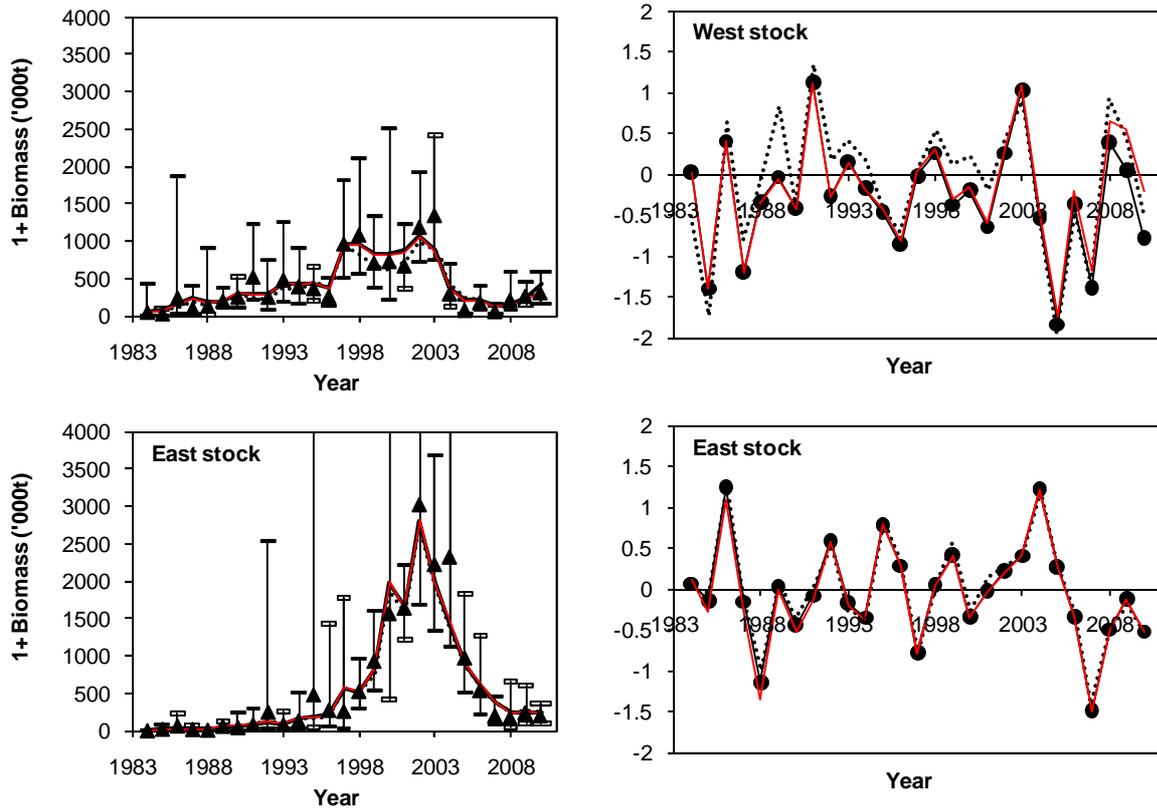


Figure 1. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2010 for Hypothesis 1 of two discrete stocks with alternatives i) excluding the prior on the recruitment residuals (thick black line), ii) higher upper bound on k_{cov}^S (dotted line), and iii) including the prior on the recruitment residuals (thin red line). The observed indices are shown with 95% confidence intervals. The standardised residuals from the fits are given in the right hand plot.

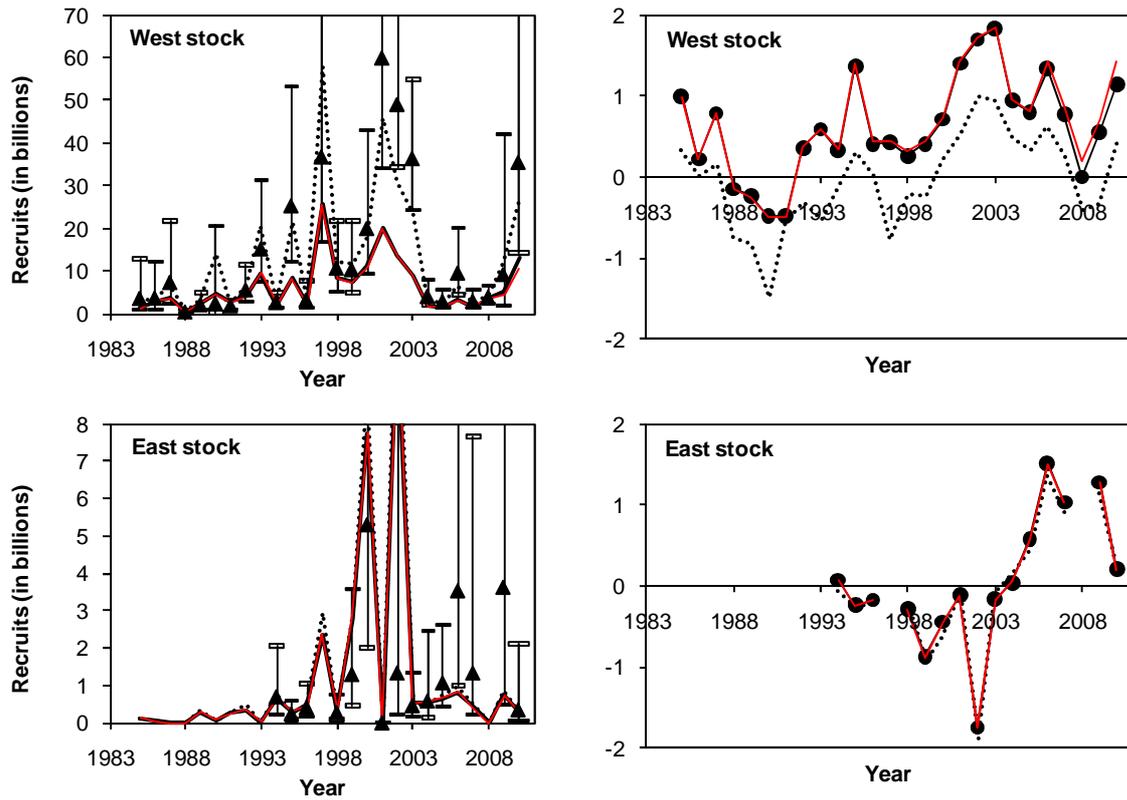


Figure 2. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2010 for Hypothesis 1 of two discrete stocks with alternatives i) excluding the prior on the recruitment residuals (thick black line), ii) higher upper bound on k_{cov}^S (dotted line), and iii) including the prior on the recruitment residuals (thin red line). The observed indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

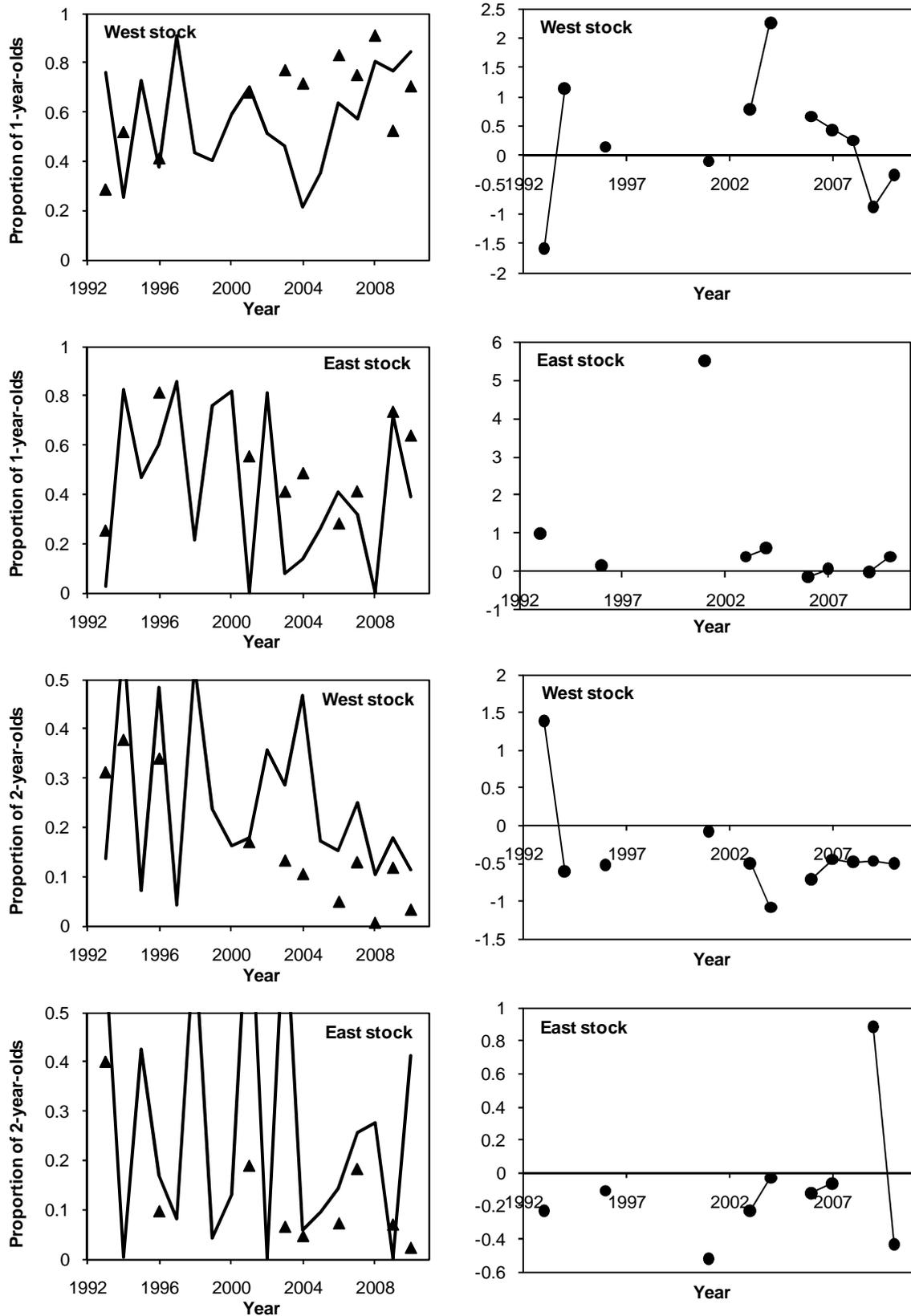


Figure 3. Observed and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the November surveys from 1993 to 2010 for Hypothesis 1 of two discrete stocks with alternative i) excluding the prior on the recruitment residuals. The residuals from the fits are given in the right hand plots.

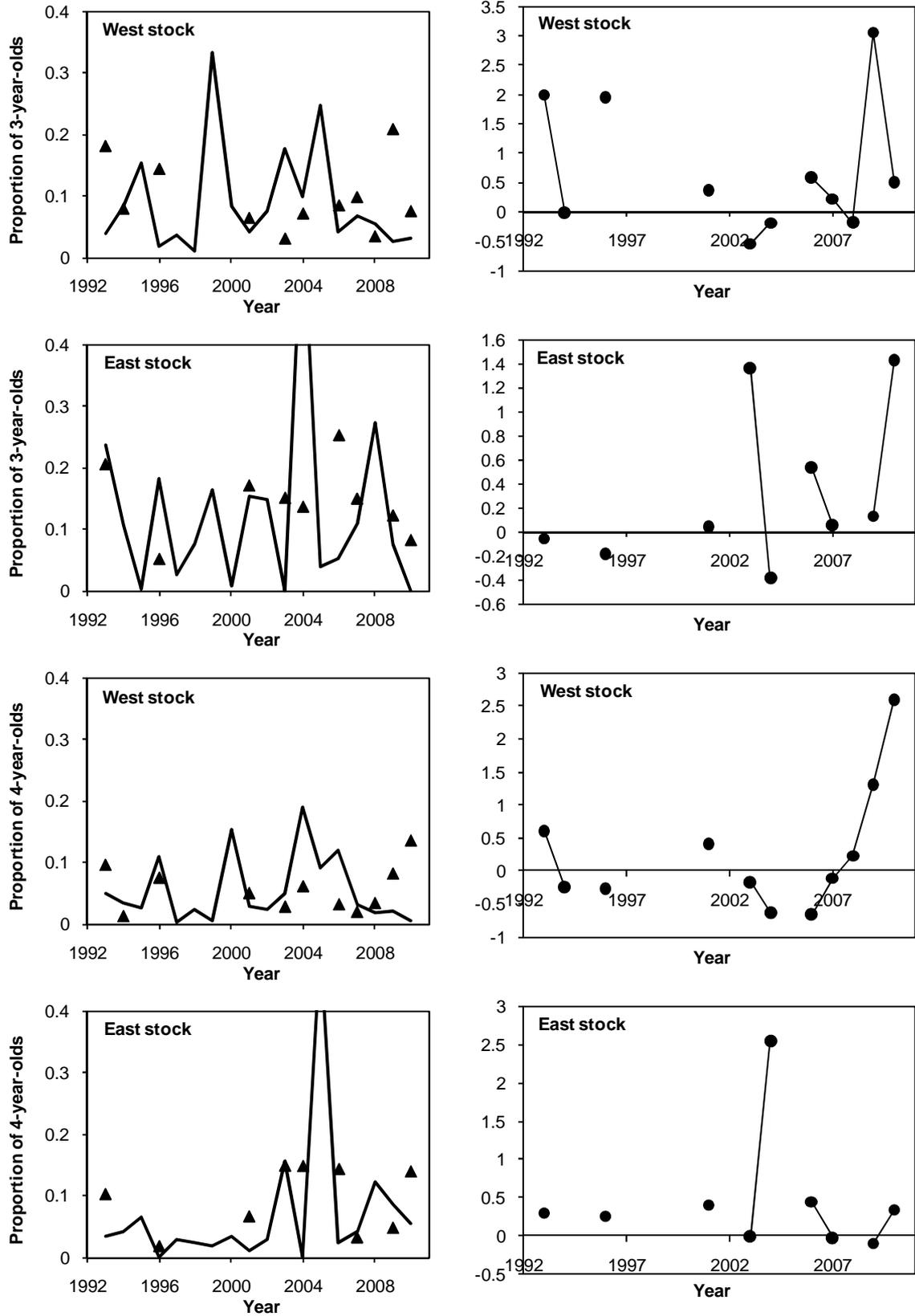


Figure 3 (continued).

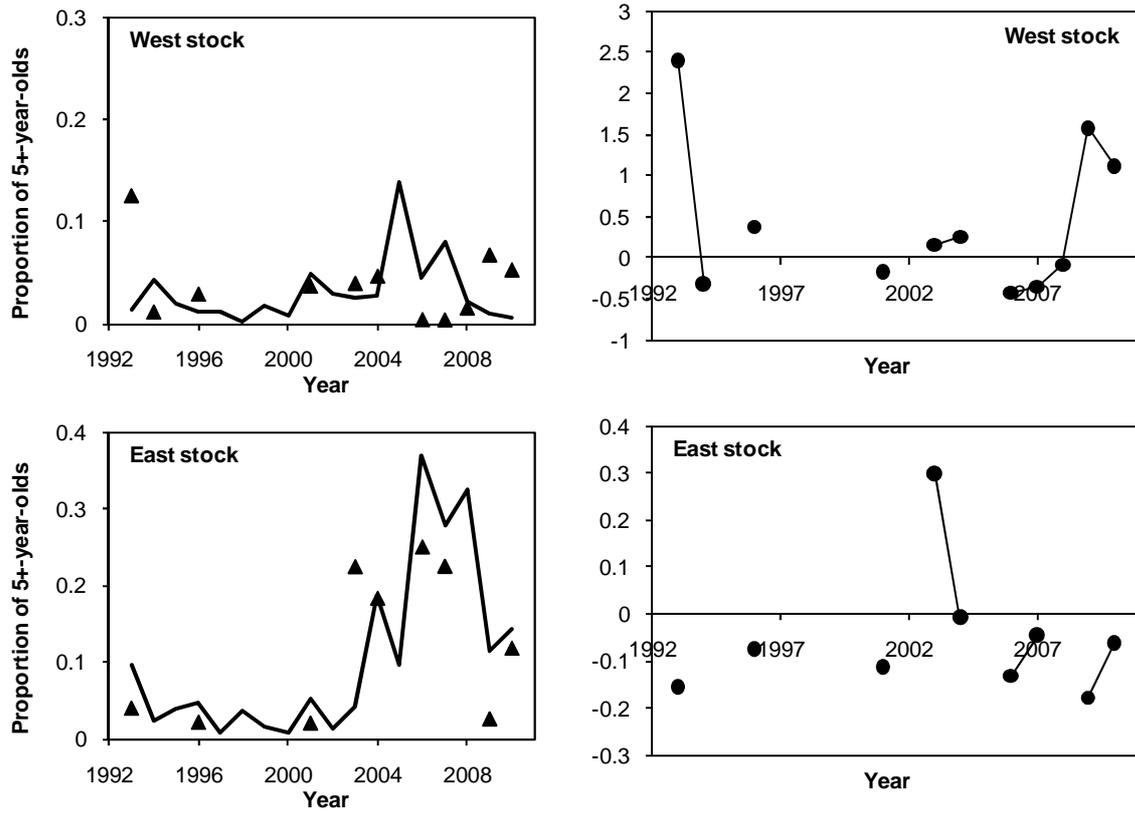


Figure 3 (continued).

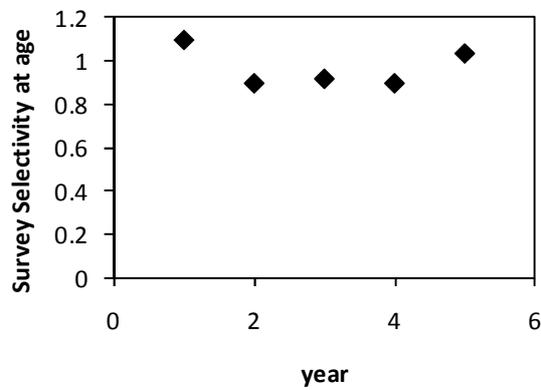


Figure 4. The model estimated November survey selectivity at age for Hypothesis 1 of two discrete stocks with alternative i) excluding the prior on the recruitment residuals.

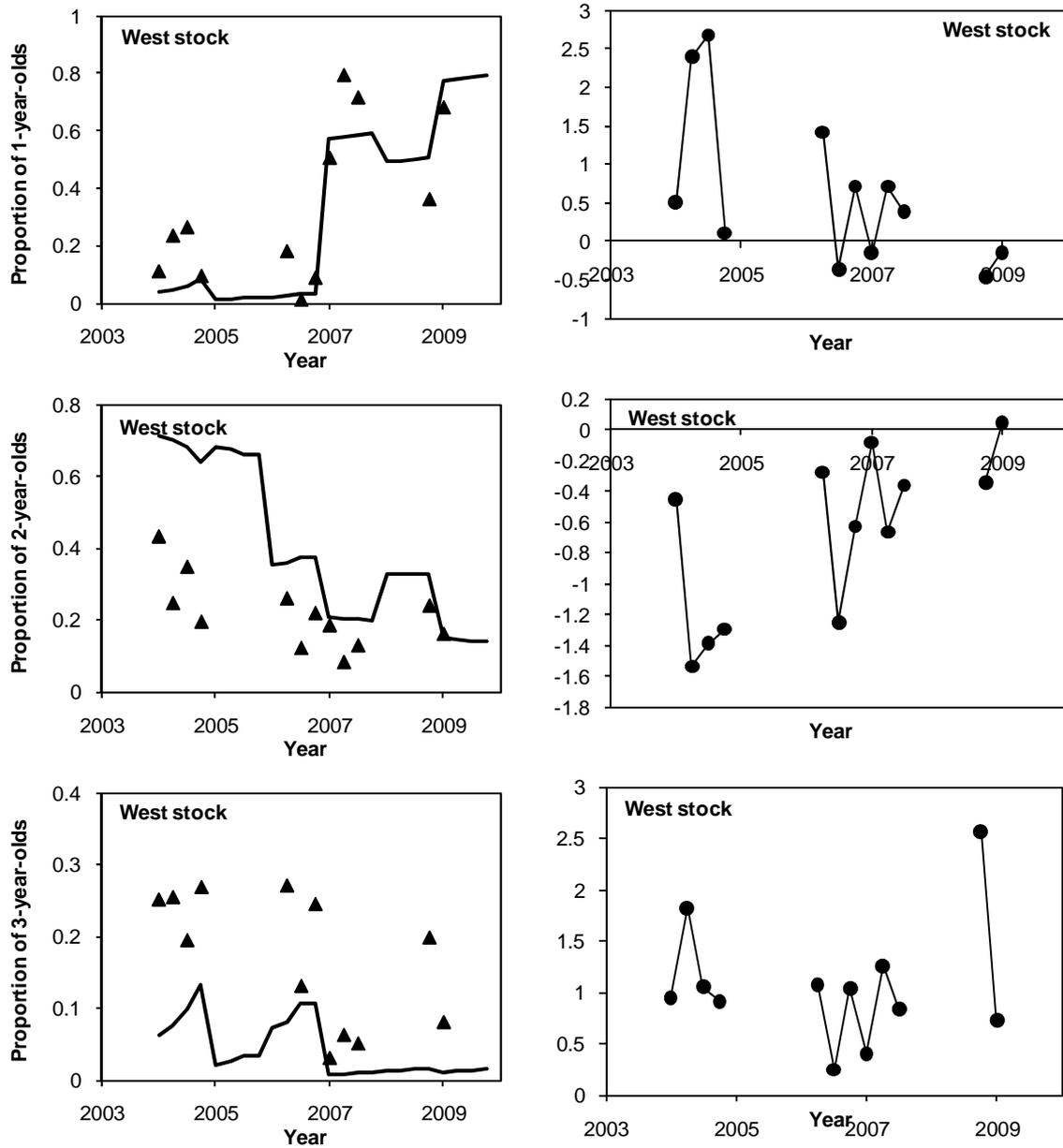


Figure 5. Observed and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the quarterly commercial catch from 2004 to 2009 for Hypothesis 1 of two discrete stocks with alternative i) excluding the prior on the recruitment residuals. The residuals from the fits are given in the right hand plots.

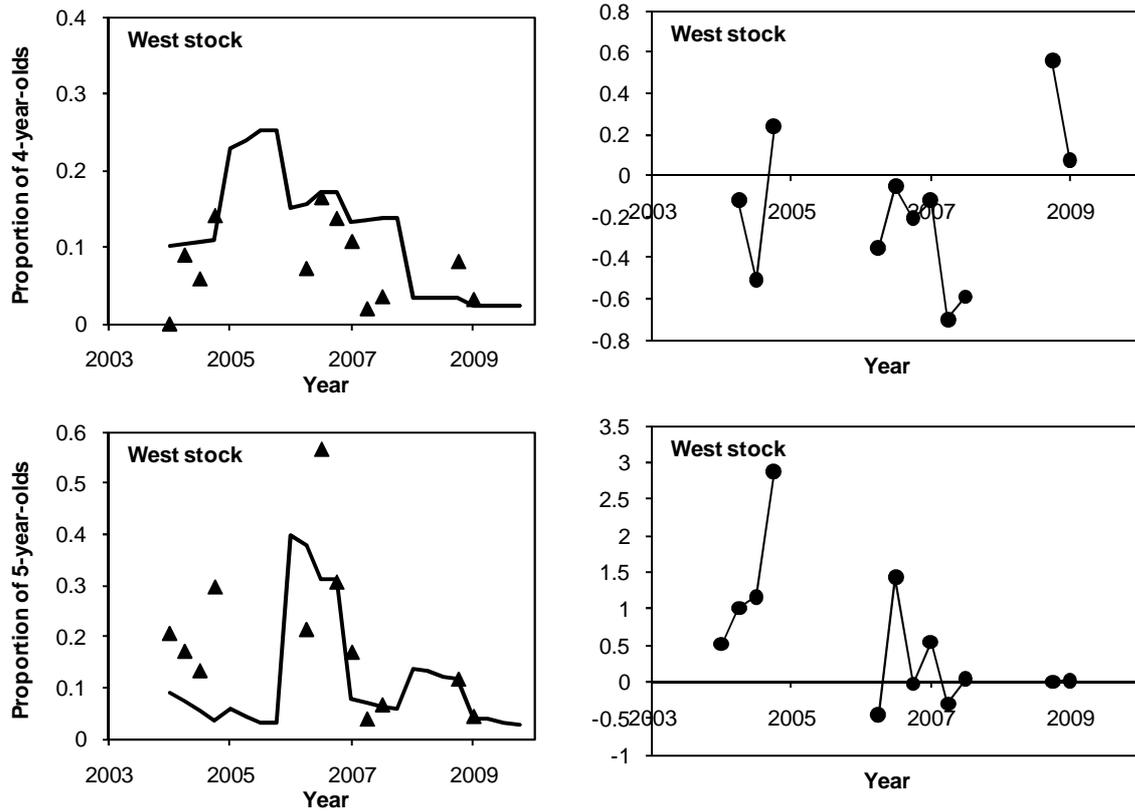


Figure 5 (continued).

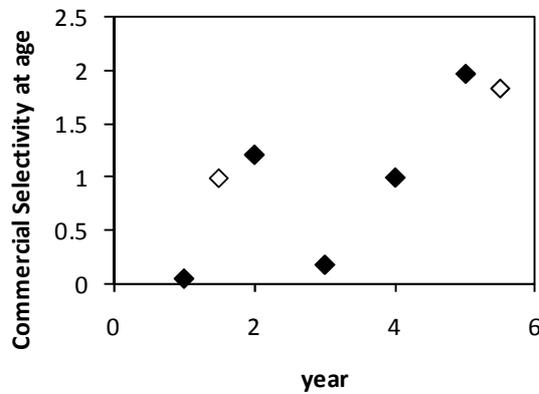


Figure 6. The model estimated commercial selectivity at age for proposed hypothesis 1 of two discrete stocks with alternatives i) excluding the prior on the recruitment residuals. The open diamonds represent the selectivity at ages 1 and 5+ estimated from 2007 to 2010, while the solid diamonds for these ages represent the selectivity from 1984 to 2006.

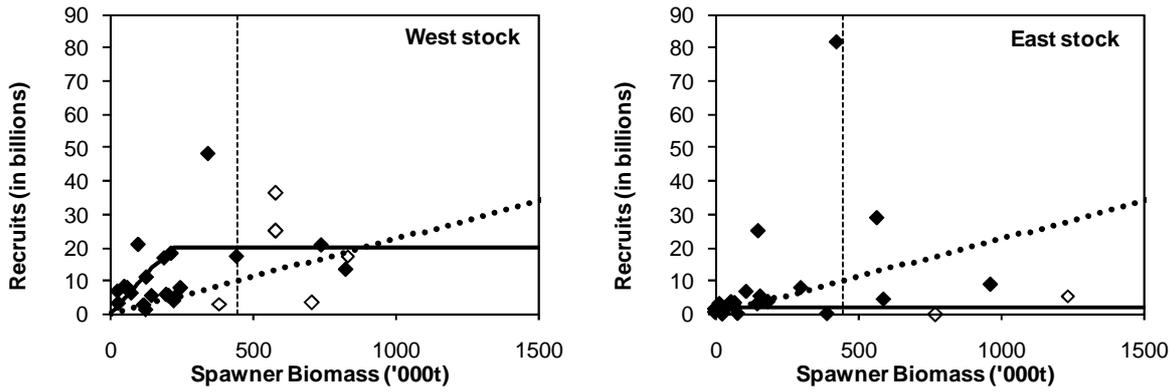


Figure 7. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2009, with the ‘hockey-stick’ stock recruitment curve and the constant recruitment between 2000 and 2004 also shown, for Hypothesis 1 with two discrete stocks with alternative iii) including the prior on the recruitment residuals. The open diamonds denote the 2000 to 2004 November spawner biomass and recruitment. The vertical thin dashed line indicates the average 1991 to 1994 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line for normal recruitment. The standardised residuals from the fits are given in the lower plots, against year and against spawner biomass for $\bar{M}_{ad}^S = 0.8$ (black) and $\bar{M}_{ad}^S = 0.6$ (red).

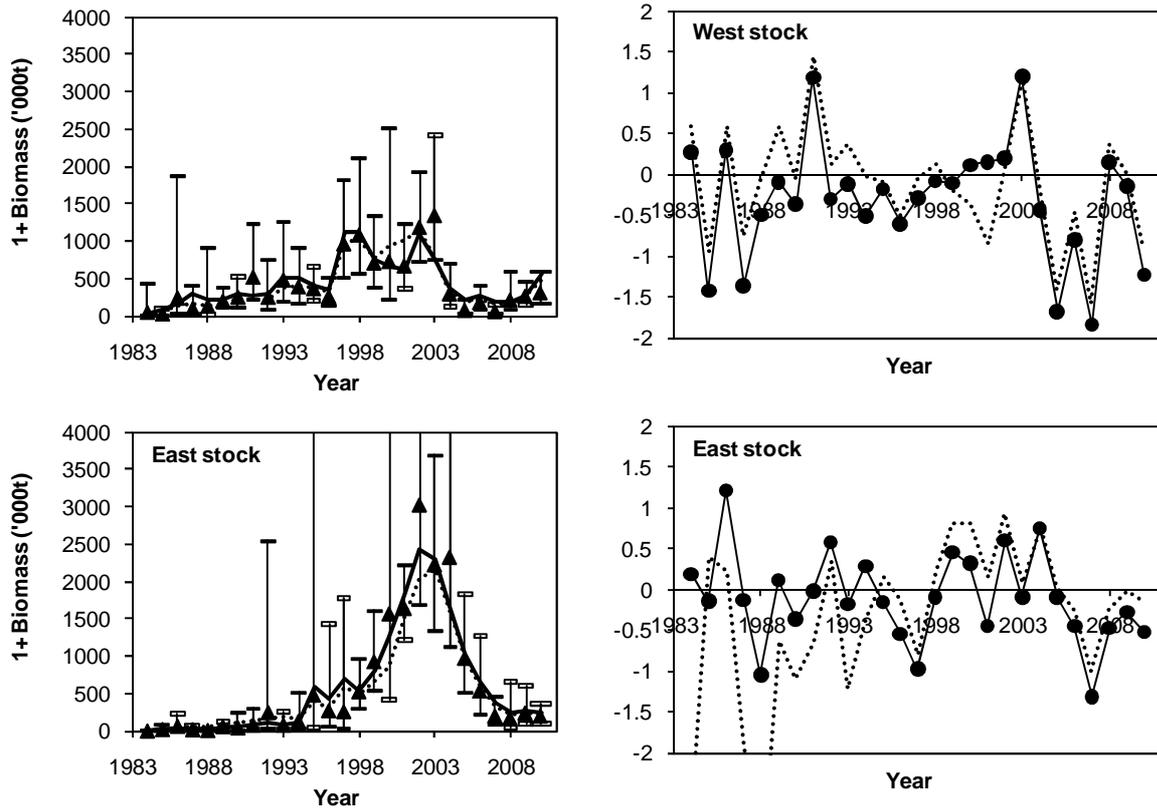


Figure 8. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2010 for Hypothesis 2 of two stocks with eastward movement of recruits for alternatives i) no movement from 1984-1993 (thick black line), and ii) average movement from 1994-2010 used prior to 1994 (dotted line). The observed indices are shown with 95% confidence intervals. The standardised residuals from the fits are given in the right hand plot.

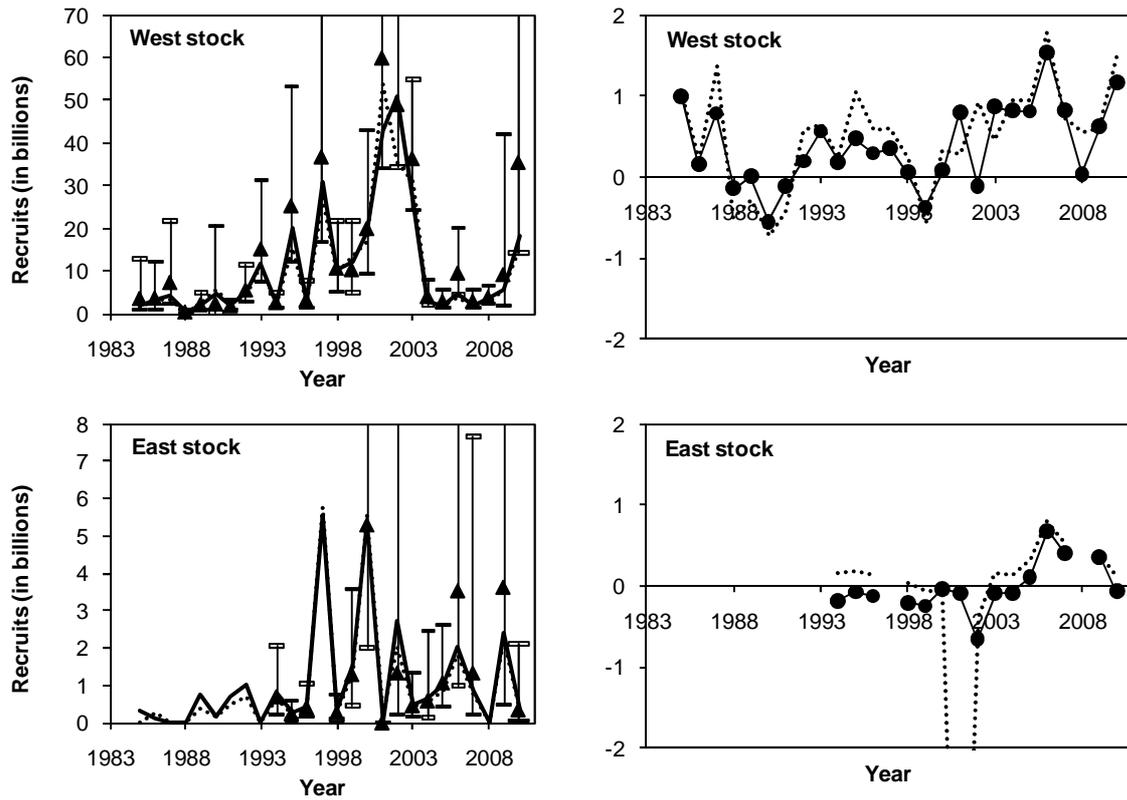


Figure 9. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2010 for Hypothesis 2 of two stocks with eastward movement of recruits for alternatives i) no movement from 1984-1993 (thick black line), and ii) average movement from 1994-2010 used prior to 1994 (dotted line). The observed indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

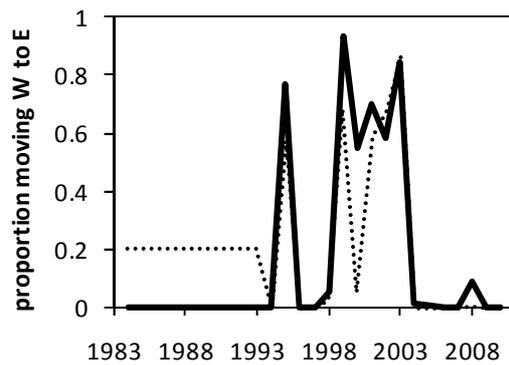


Figure 10. Time series of estimated proportion of west stock recruits which migrate to the east stock at the beginning of November for Hypothesis 2 of two stocks with eastward movement of recruits for alternatives i) no movement from 1984-1993 (thick black line), and ii) average movement from 1994-2010 used prior to 1994 (dotted line).

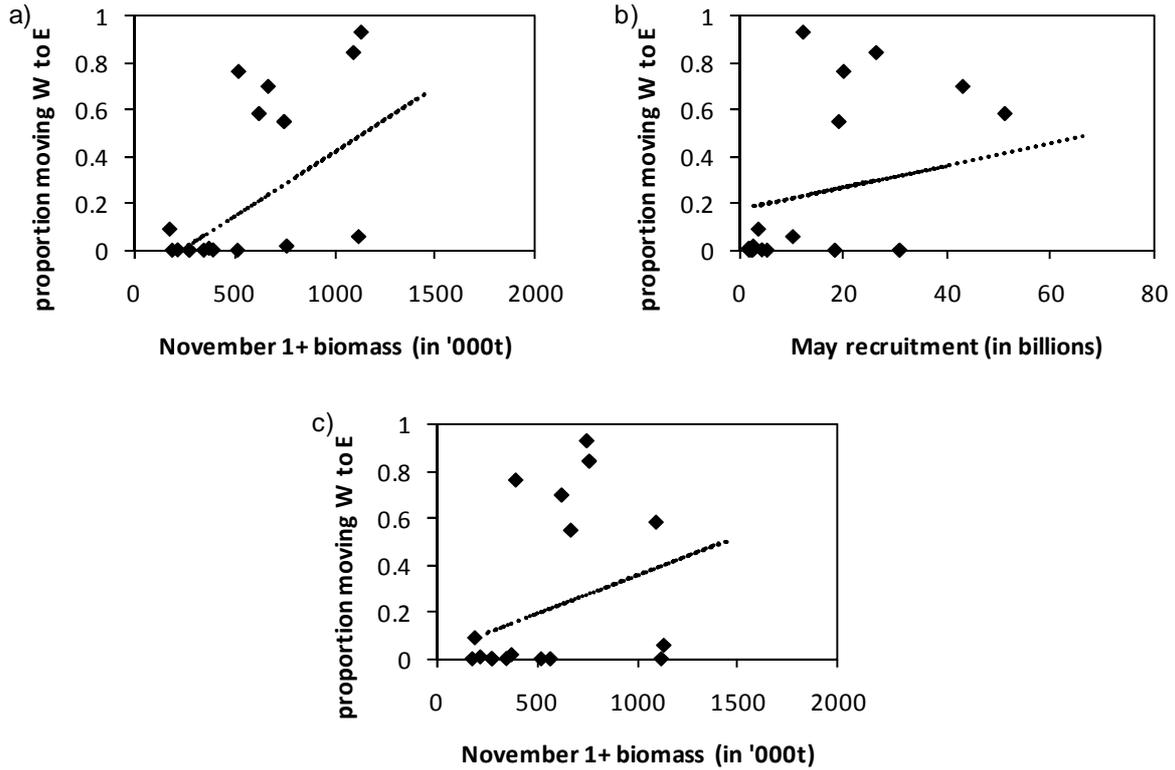


Figure 11. The model predicted proportion of west stock recruits which migrate to the east stock at the beginning of November plotted against a) November 1+ biomass of the previous year, b) May recruitment of the current year, and c) November 1+ biomass of the current year, for Hypothesis 2 of two stocks with eastward movement of recruits for alternative i) no movement from 1984-1993. The straight line regressions between the two sets of data in each plot are also shown, with correlation coefficients of a) 0.40, b) 0.35 and c) 0.11.

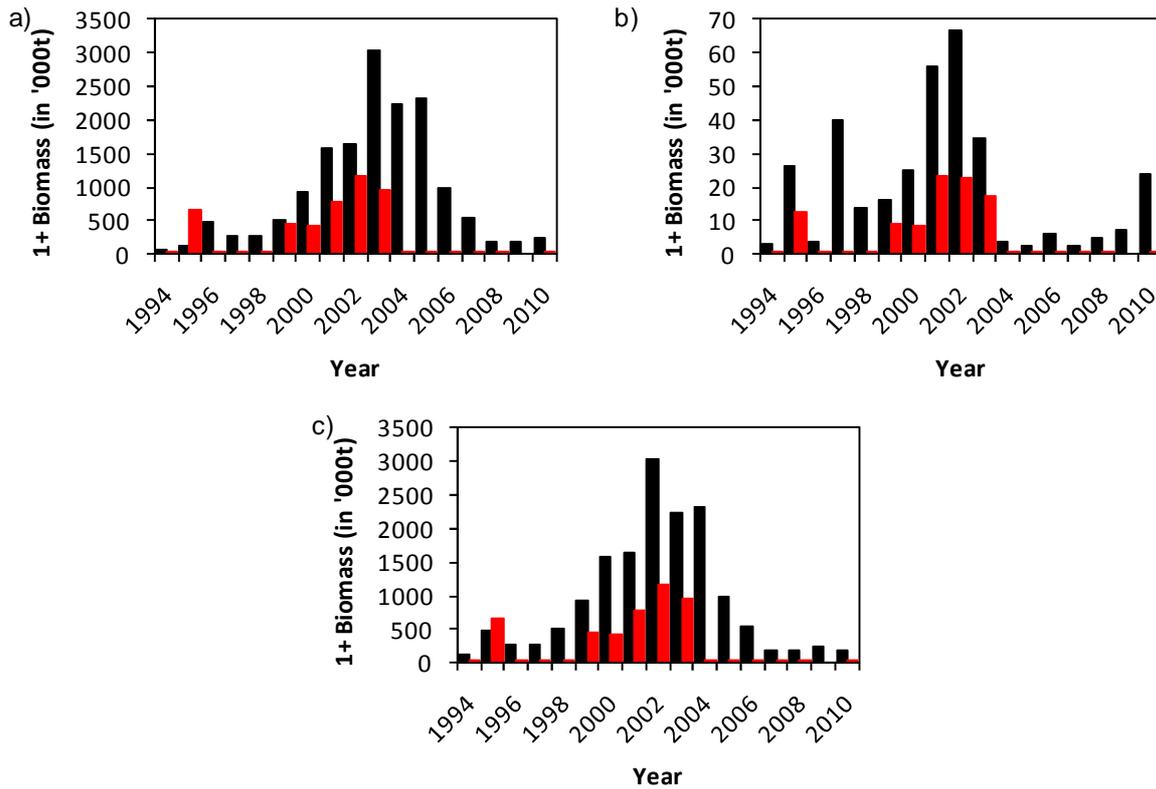


Figure 12. The model predicted biomass/numbers of west stock recruits which migrate to the east stock at the beginning of November (red) plotted with a) November 1+ biomass of the previous year, b) May recruitment of the current year, and c) November 1+ biomass of the current year (in black), for Hypothesis 2 of two stocks with eastward movement of recruits for alternative i) of no movement from 1984-1993.

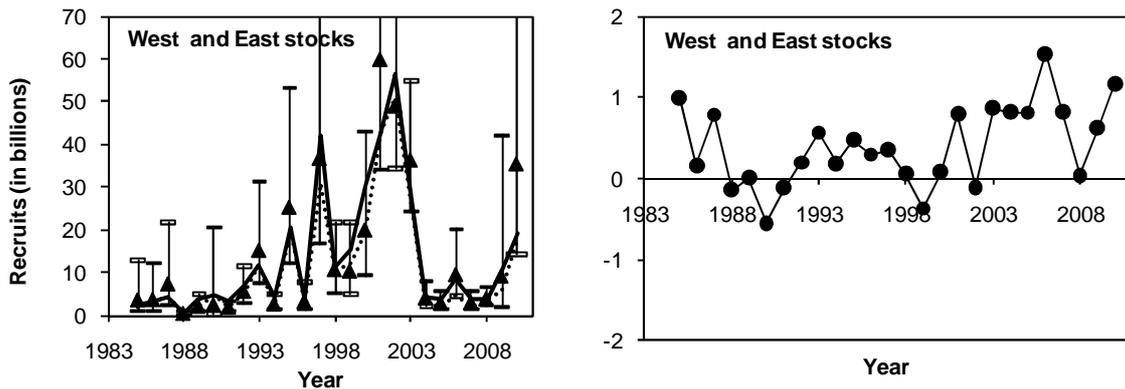


Figure 13. Acoustic survey estimated and model predicted total sardine recruitment numbers from May 1985 to May 2010 for Hypothesis 3. The model predicted sardine recruitment numbers for the west stock only are also shown (dotted line). The observed indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

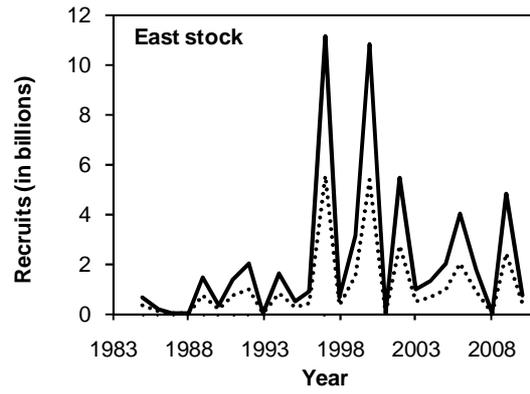


Figure 14. Model predicted sardine recruitment numbers from the east stock from May 1985 to May 2010 for Hypothesis 3 (solid line) and Hypothesis 2, alternative i) (dotted line).

Appendix: Glossary of parameters used in this document

Annual numbers and biomass:

- $N_{j,y,a}^S$ - model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j
- $B_{j,y}^S$ - model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y , associated with the November survey
- $SSB_{j,y}^S$ - model predicted spawning stock biomass (in thousand tonnes) of stock j at the beginning of November in year y
- $w_{j,y,a}^S$ - mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y
- $N_{j,y,r}^S$ - model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y
- t_y^S - time lapsed (in months) between 1 May and the start of the recruit survey in year y
- $move_y$ - proportion of west stock recruits which migrate to the east stock at the beginning of November of year y

Natural mortality:

- $M_{a,y}^S$ - rate of natural mortality (in year⁻¹) of sardine of age a in year y
- \bar{M}_{ju}^S - median juvenile rate of natural mortality (in year⁻¹)
- \bar{M}_{ad}^S - median adult rate of natural mortality (in year⁻¹)
- ϵ_y^{ad} - annual residuals about adult natural mortality
- η_y^{ad} - normally distributed error used in calculating ϵ_y^{ad}
- σ_{ad} - standard deviation in the annual residuals about adult natural mortality
- p - annual autocorrelation coefficient in annual residuals about adult natural mortality

Catch:

- $C_{j,y,a,q}^S$ - model predicted number (in billions) of sardine of age a of stock j caught during quarter q of year y
- $C_{j,y,m,l}^{RLF}$ - number of fish in length class l landed in month m of year y of stock j (the 'raised length frequency')
- $lcut_{y,m}$ - cut off length for recruits in month m of year y

$C_{j,y,q,a}^{bycatch}$ - the number of fish of age $a \geq 1$ from the anchovy-directed fishery in quarter q of year y

$S_{j,y,q,a}$ - commercial selectivity at age a during quarter q of year y of stock j

$F_{j,y,q}$ - fished proportion in quarter q of year y for a fully selected age class a of stock j , by the directed and redeye bycatch fisheries

$\tilde{C}_{j,y,Obs}^S$ - number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey

Proportions at age:

$P_{j,y,a}^S$ - model predicted proportion-at-age a of stock j in the November survey of year y

$S_{j,a}^{survey}$ - survey selectivity at age a in the November survey for stock j

$P_{j,y,q,a}^{com,S}$ - model predicted proportion-at-age a of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

Recruitment:

a_j^S - maximum recruitment of stock j (in billions)

b_j^S - spawner biomass above which there should be no recruitment failure risk in the hockey stick model for stock j

c^S - constant recruitment (distribution median) during the “peak” years of 2000 to 2004

$\mathcal{E}_{j,y}^S$ - annual lognormal deviation of sardine recruitment.

$\sigma_{j,r}^S$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve of stock j

$\sigma_{r,peak}^S$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve during peak years in the single stock hypothesis

Proportions at length and growth curve:

$P_{j,y,l}^S$ - model predicted proportion-at-length l of stock j associated with the November survey in year y

$A_{j,a,l}^{sur}$ - proportion of sardine of age a of stock j that fall in the length group l in November

$P_{j,y,q,l}^{coml,S}$ - model predicted proportion-at-length l of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

$A_{j,q,a,l}^{com}$ - proportion of sardine of age a of stock j that fall in the length group l in quarter q

$L_{j,\infty}$ - maximum length of sardine of stock j

- κ_j - annual growth rate of sardine of stock j
- $t_{0,j}$ - age at which the growth rate is zero of sardine of stock j
- $\vartheta_{j,a}$ - variance about the meal length for age a of sardine of stock j

Likelihoods:

- $\ln L^{Nov}$ - contribution to the negative log likelihood from the model fit to the November 1+ biomass data
- $\ln L^{rec}$ - contribution to the negative log likelihood from the model fit to the May recruit data
- $\ln L^{sur\ propa}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-age data
- $\ln L^{com\ propa}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-age data
- $\ln L^{sur\ propl\ min}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only
- $\ln L^{sur\ propl}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only
- $\ln L^{com\ propl\ min}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the minus length class only
- $\ln L^{com\ propl}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the remaining length classes
- $\hat{B}_{j,y}^S$ - acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y
- $\sigma_{j,y,Nov}^S$ - survey sampling CV associated with $\hat{B}_{j,y}^S$ that reflects survey inter-transect variance
- $k_{j,N}^S$ - constant of proportionality (multiplicative bias) associated with the November survey of stock j
- k_{ac}^S - multiplicative bias associated with the acoustic survey
- $\hat{N}_{j,y,r}^S$ - acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y
- $\sigma_{j,y,rec}^S$ - survey sampling CV associated with $\hat{N}_{j,y,r}^S$ that reflects survey inter-transect variance
- $k_{j,r}^S$ - constant of proportionality (multiplicative bias) associated with the recruit survey of stock j
- k_{cov}^S - multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey

- k_{covE}^S - multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey
- $(\lambda_{j,N/r}^S)^2$ - additional variance (over and above $\sigma_{y,Nov/rec}^S$) associated with the November/recruit surveys of stock j
- $\hat{p}_{j,y,a}^S$ - estimate of the proportion (by number) of sardine of age a in stock j in the November survey of year y
- $n_{s,y}$ - number of fish from the November survey trawls in year y used to compile the age-length key for calculating $\hat{p}_{j,y,a}^S$
- $(\sigma_p^S)^2$ - overall variance-related parameter for the log-transformed survey proportion-at-age observations, $\hat{p}_{j,y,a}^S$ [note variance = $(\sigma_p^S)^2 / (n_y \hat{p}_{j,y,a}^S)$]
- ys - years for which ALKs are available to calculate proportion-at-age in the November survey ('93, '94, '96, '01, '03, '04, '06-'10);
- w_{propa}^{survey} - weighting applied to the survey proportion-at-age data
- $\hat{p}_{y,q,a}^{com,S}$ - estimate of the proportion (by number) of single-stock or "west stock" sardine of age a in the commercial catch of quarter q of year y
- $n_{y,q}^{com}$ - number of fish from the commercial trawls in quarter q of year y used to compile the age-length key for calculating $\hat{p}_{y,q,a}^{com,S}$
- $(\sigma_{com}^S)^2$ - overall variance-related parameter for the log-transformed commercial proportion-at-age observations, $\hat{p}_{y,q,a}^{com,S}$ [note variance = $(\sigma_{com}^S)^2 / (n_{y,q}^{com} \hat{p}_{y,q,a}^{com,S})$]
- yc / qc - years/quarters for which ALKs are available to calculate quarterly proportions-at-age in the commercial catch ('04 Q1-4, '06 Q2-4, '07 Q1-3, '08 Q4, '09 Q1);
- w_{propa}^{com} - weighting applied to the commercial proportion-at-age data
- $\hat{p}_{j,y,q,l}^{coml,S}$ - observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q of year y ;
- $w_{proplmin}^{com}$ - weighting applied to the commercial proportion at length data for the minus length class
- w_{propl}^{com} - weighting applied to the remaining commercial proportion at length data
- $\sigma_{comlmin}^S$ - variance-related parameter for the log-transformed commercial proportion-at-length data of the minus length class
- σ_{coml}^S - variance-related parameter for the log-transformed commercial proportion-at-length data

Other:

F_{init} - rate of fishing mortality assumed in the initial year

$s_{j,cor}^S$ - recruitment serial correlation for stock j

$\eta_{j,2009}^S$ - standardised recruitment residual value for 2009 for stock j

K_j^S - carrying capacity for stock j

K_{peak}^S - carrying capacity during peak years (only for single stock hypothesis)

$\bar{w}_{j,a}^S$ - mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available